SU(3) Symmetry and the Nonleptonic K-Meson Processes*

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Consequences of the SU(3) octet scheme for the nonleptonic weak interactions are studied in some of the nonleptonic K-meson processes. The $K_1^0-K_2^0$ mass difference and the rates of $K_2^0\to 2\gamma$ and $K_2^0\to\pi^++\pi^-+\gamma$ decays are discussed. The effect of $\omega-\varphi$ mixing is considered in the $K_2^0\to 2\gamma$ decay. The possible violation of the $|\Delta I|=\frac{1}{2}$ rule in the $K_2^0\to 3\pi$ decay is discussed in terms of the η-meson pole contribution. The characteristic features of $K^+\to\pi^++\pi^0+\gamma$ and $K^+\to 3\pi$ decays in our model are also discussed. It is inferred that the rate of $K^+\to\pi^++\pi^0+\gamma$ decay is dominated by the internal bremsstrahlung contribution. A possible effect of a unitary singlet pseudoscalar meson on these problems is also discussed.

I. INTRODUCTION

RECENTLY, many attempts have been made to understand the transformation properties of the weak interaction within the framework of the SU(3) symmetry theory.[1–4] By assigning simple SU(3) transformation properties, either to the baryon and meson currents and/or to the weak Lagrangian as a whole, various authors have proposed schemes which reproduce the experimentally established selection rules. As the simplest higher symmetry scheme which contains in itself the $|\Delta I|=\frac{1}{2}$ rule for the nonleptonic processes, one may propose that the nonleptonic weak-interaction Lagrangian transforms as a member of an octet of the SU(3) group. We shall not be concerned here about the origin of this transformation property of nonleptonic interactions. At any rate, by introducing the neutral as well as the charged currents, it is possible to construct the nonleptonic interaction of current-current type which behaves like the member of SU(3) octet.[5,10] One may also speculate that some mechanism selectively enhances the contribution of octuplet channel even if the basic interactions do not belong to the SU(3) octet.[5]

The consequences of this proposal may be tested in the hyperon decays.[6–11] For the S-wave amplitude, it gives a sum rule which is consistent with experiments. It is also interesting to study the nonleptonic decays of the K meson from this standpoint, since the SU(3) relation fixes the relation between the weak vertices $K\pi$ and $K\eta$. Some of the results have already been obtained in the processes in which these vertices are likely to play an important dynamical role.[12,13] In this paper we would like to discuss this problem in a more coherent and unified way in order to obtain a more direct insight into the dynamics of nonleptonic processes. In Sec. II we explain our model and discuss the K_1^0-K_2^0 mass difference. The $K_2^0\to 2\gamma$ decay is treated in Sec. III by including the effect of $\omega-\varphi$ mixing. In Sec. IV, the $K_2^0\to\pi^++\pi^-+\gamma$ and $K^+\to\pi^++\pi^0+\gamma$ decay rates will be calculated. In Sec. V, some comments will be made about the dynamics of $K\to 3\pi$ decay, and the possible violation of the $|\Delta I|=\frac{1}{2}$ rule in the $K_2^0\to 3\pi$ decay, due to the η-meson pole contribution. In Sec. VI, further remarks about the K_1^0-K_2^0 mass difference and the dynamics of nonleptonic processes will be added.

II. DYNAMICAL MODEL AND THE $K_1^0-K_2^0$ MASS DIFFERENCE

Our aim is to exploit the octuplet nature of nonleptonic weak interactions in studying the processes in which the η meson as well as the π meson plays an important intermediary role. As a dynamical model for the processes under consideration, we shall use the one-pole approximation involving the pseudoscalar meson. According to our assumption, the effective Lagrangian for the two-body weak transition between pseudoscalar mesons transforms like λ_6.

$$\mathcal{L}_W=-\sqrt{2}m_K^2 f_W\,\mathrm{Tr}(\lambda_6 PP)\,, \tag{1}$$

where P represents the 3×3 matrix corresponding to the octet pseudoscalar meson, and the λ_6 is the sixth component in the unitary spin space.[2] Then $\mathcal{L}_W$ can be

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[12] S. Hori, Progr. Theoret. Phys. (Kyoto) **29**, 612 (1963); S. Oneda and S. Hori, Phys. Rev. **132**, 1800 (1963); see summary (V) where the SU(3) octet model and some of its implications on K-meson processes are discussed even earlier than Refs. 6–11. The sign of $\Delta m(K^0)$ is in error and must be changed throughout the paper. Similar results were also obtained by S. N. Biswas and S. K. Bose, Phys. Rev. Letters **12**, 177 (1964).
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expressed as[12,13]

$$\mathcal{L}_W = -\sqrt{2} m_K{}^2 f_W \times\{(K^+\pi^- + K^-\pi^+) - K_2{}^0(\pi^0 + 3^{-1/2}\eta^0)\}. \quad (2)$$

We estimate the coupling constant f_W from the observed $K_1{}^0$-$K_2{}^0$ mass difference

$$\Delta m(K^0) = m(K_1{}^0) - m(K_2{}^0).$$

Now, by assuming that the π^0 and η^0 poles yield the principal contributions to this mass difference, we obtain[11]

$$\Delta m(K^0) = -f_W{}^2 m_{K^0} \times\left\{\frac{m_{K^0}{}^2}{m_{K^0}{}^2 - m_\pi{}^{02}} + \frac{m_{K^0}{}^2}{3(m_{K^0}{}^2 - m_\eta{}^{02})}\right\}. \quad (3)$$

Note that $\Delta m(K^0)$ obtained in (6) is proportional to $4m_K{}^2 - 3m_\eta{}^2 - m_\pi{}^2$, which will be zero if we use the Gell-Mann–Okubo first-order mass formula. Experimental values of $\Delta m(K^0)$ have not yet been comfortably settled, and their values range from 0.5 to 1.5 $\hbar/\tau(K_1{}^0)$.[14] We here adopt the following value, taking into account its present experimental uncertainty:

$$|\Delta m(K^0)| = 1\times(1\pm0.5)[\hbar/\tau(K_1{}^0)] = 0.65(1\pm0.5)\times10^{-11}\ \text{MeV}.$$

Then we obtain from (3)

$$f_W{}^2 = 3.1\times(1\pm0.5)\times10^{-14}, \quad (4)$$

and the sign of mass difference[15] is such that

$$m(K_1{}^0) > m(K_2{}^0). \quad (5)$$

The π-η pole approximation which we have used above needs some justification. First, the most important competing contributions could be expected from the S-wave two-pion intermediate states. In particular, if the so-called dipion[16] resonance σ^0 exists, it may contribute significantly to the mass difference. In fact, a possibility that the dipion contribution dominates this mass difference, the rate of $K_1{}^0 \to 2\pi$ decay, and the low-energy S-wave pion-pion scattering at the same time has been pointed out by Oneda *et al.*,[17] and also by Nishijima.[18] Note that we obtain $m(K_1{}^0) \gtrless m(K_2{}^0)$ according as $m_{\sigma^0} \lesseqgtr m_{K^0}$.

However, Gell-Mann has recently shown that the assumptions (a) CP invariance of weak interaction, and (b) current-current type of nonleptonic interaction transforming like an octet (either effectively or exactly) with the currents which also belong to an octet, will cause the $K_1{}^0 \to 2\pi$ decay to be forbidden.[19] If we adopt this model, the parity-violating part of weak Lagrangian has an opposite charge-conjugation parity to the $K_1{}^0$ meson, so that a $K_1{}^0 \to \sigma^0$ transition is forbidden. Likewise, the $K_1{}^0 \to 2\pi$ decay through $K_1{}^0 \to \sigma^0 \to 2\pi$ is not allowed. Thus, in this model one can neglect the contribution of the dipion to the $K_1{}^0$-$K_2{}^0$ mass difference. Also, the existence of dipion has not yet been clearly established. Second, we have to worry about the contribution of vector-meson resonance states. Of course, we have a similar relation to (2) for the K-meson–vector-meson coupling, i.e.,

$$-f_{K^+\rho^+} = f_{K_2{}^0\rho^0} = \sqrt{3} f_{K_2{}^0\omega_8},$$

where ω_8 belongs to the $I=Y=0$ member of the bare vector-meson octet which must be expressed in terms of the physical ω and φ particles with a mixing parameter. Now these vector-meson contributions do not behave as a pole, contrary to the case of pseudoscalar meson, and they always contribute negative value to $\Delta m(K^0)$. However, the vertices have a higher momentum barrier than the $K-\pi(\eta)$ vertex, so that offhand, the contribution is expected to be suppressed by this barrier effect. In fact, a simple-minded but plausible computation indicates that the contribution of the ρ meson, for instance, is only around 2% of the observed value,[20] while the same type of calculation[17] for the $K_2{}^0\pi$ vertex yields a value which is very close to the one obtained in (4). We therefore feel that our approximation for the

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[18] K. Nishijima, Phys. Rev. Letters **12**, 41 (1964).

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[20] S. Oneda (unpublished). At present this seems to be a *controversial* point. For instance, in the hyperon decays there is a contribution due to $K^*-\pi$ vertex for the parity-violating amplitude. Since we do not have an absolutely reliable estimate of this vertex, we may assume that the parity-violating amplitude is dominated by the $K^*-\pi$ diagram. See, for instance, B. W. Lee and A. R. Swift, A dynamical basis of the sum rule $2\Xi_-{}^- = \Lambda_- + \sqrt{3}\Sigma_0{}^+$ (to be published). Attempts along this line have also been pursued by S. Hori *et al.* (private communication). If we assume that the $K^*-\pi$ vertex is so large as to be responsible for the parity-violating amplitude of hyperon decay, the $K_1{}^0$-$K_2{}^0$ mass difference will also receive a significant contribution from the vector meson intermediate states. [They contribute to $\Delta m(K^0) < 0$.] Furthermore, the $K_1{}^0 \to 2\pi$ decay rate also turns out to be explained by the process $K^0 \to K^* + \pi \to \pi + \pi$ through $K^*-\pi$ diagram. However, if we use our estimate of $K^*-\pi$ vertex mentioned above, the contribution of $K^*-\pi$ vertex to hyperon decay is only a few percent, and we were not able to convince ourselves that the $K^*-\pi$ contribution is so important. The same point of view was also expressed by J. C. Pati, Phys. Rev. **130**, 2097 (1963), footnote 14.

K_1^0-K_2^0 mass difference seems rather reasonable, especially if we take the Gell-Mann model and can neglect the dipion contribution.

In the following discussion, we thus use the value obtained in (4) as a reasonable measure of the magnitude of f_W. We add more comments on the K_1^0-K_2^0 mass difference in Sec. VI.

III. $K_2^0 \to 2\gamma$ DECAY

We now use the dynamical model to calculate the $K_2^0 \to 2\gamma$ decay rate. In this model, the K_2^0 meson first goes to a $\pi^0(\eta^0)$ meson through the weak Lagrangian of Eq. (2), and the $\pi^0(\eta^0)$ meson decays into the two final-state photons. From the SU(3) transformation properties of electromagnetic interactions, we have the relation

$$M(\eta^0 \to 2\gamma) = 3^{-1/2} M(\pi^0 \to 2\gamma), \tag{6}$$

where $M(\eta^0 \to 2\gamma)$ and $M(\pi^0 \to 2\gamma)$ are, respectively, the $\eta^0 \to 2\gamma$ and $\pi^0 \to 2\gamma$ decay matrix elements.

The decay rate is then

$$P(K_2^0 \to 2\gamma) \approx f_W^2 \left[\frac{m_K^2}{m_\pi^2 - m_K^2} + \frac{m_K^2}{3(m_\eta^2 - m_K^2)}\right]^2 \times \left(\frac{m_K}{m_\pi}\right)^3 P(\pi^0 \to 2\gamma). \tag{7}$$

Once again, the π and η pole terms tend to cancel each other, leading to a small decay rate. Now, using the latest experimental value of the $\pi^0 \to 2\gamma$ decay rate,[21,22]

$$1/P(\pi^0 \to 2\gamma) = (1.05 \pm 0.18) \times 10^{-16}\ \text{sec}, \tag{8}$$

we find[12]

$$P(K_2^0 \to 2\gamma) \approx 0.76(1 \pm 0.5) \times 10^4\ \text{sec}^{-1}. \tag{9}$$

Up to now we have used the electromagnetic coupling of the π^0 and η^0 mesons in the exact SU(3) limit. In order to take into account the symmetry-breaking interactions, we use the model[23] in which photons interact with the pseudoscalar mesons through the neutral intermediate vector mesons having their observed masses. In order to explain the mass spectra of the vector mesons, it has been proposed that the observed ω and φ mesons are linear superpositions of ω_1 (a unitary singlet) and ω_8 (the $I = Y = 0$ member of a unitary octet).[24] If we write

$$|\varphi\rangle = \cos\theta|\omega_8\rangle - \sin\theta|\omega_1\rangle, \tag{10}$$
$$|\omega\rangle = \sin\theta|\omega_8\rangle + \cos\theta|\omega_1\rangle,$$

then the SU(3)-invariant vector-vector pseudoscalar-meson Lagrangian becomes[25]

$$\begin{aligned}\mathcal{L} = \pi^0\rho^0\{\omega(g\cos\theta + f\sin\theta) + \varphi(f\cos\theta - g\sin\theta)\} \\ + \eta^0\{\omega^2(g\cos\theta - \tfrac{1}{2}f\sin^2\theta) - \varphi^2(g\sin\theta\cos\theta + \tfrac{1}{2}f\cos^2\theta) \\ + \omega\varphi(g\cos 2\theta - \tfrac{1}{2}f\sin 2\theta) + \tfrac{1}{2}f\rho^0\rho^0\},\end{aligned} \tag{11}$$

where g and f are, respectively, the singlet and octet (D-type) coupling constants.

The vector-meson–photon vertices satisfy the SU(3)-type relations

$$G_{w_1\gamma} = 0, \quad G_{\rho\gamma} = \sqrt{3}G_{w_8\gamma}. \tag{12}$$

Thus,

$$M(\pi^0 \to 2\gamma) = 3^{-1/2}(G_{\rho\gamma})^2 \times \frac{1}{m_\rho^2 m_w^2}\{f + \epsilon(g\sin\theta\cos\theta - f\cos^2\theta)\},$$

$$\epsilon = (m_\varphi^2 - m_w^2)/m_w^2.$$

Similarly,

$$M(\eta^0 \to 2\gamma) = (G_{\rho\gamma}^2/m_\rho^4)(\tfrac{1}{2}f)(1-\beta),$$

where

$$\beta = \tfrac{1}{3}(m_\rho/m)^4\{1 - 2\lambda((g/f)\sin 2\theta + \cos 2\theta) + \lambda^2\cos 2\theta((2g/f)\sin 2\theta + \cos 2\theta)\}$$

and

$$\frac{2}{m^2} = \frac{1}{m_w^2} + \frac{1}{m_\varphi^2}, \quad \frac{2\lambda}{m^2} = \frac{1}{m_w^2} - \frac{1}{m_\varphi^2}.$$

The rate of the $K_2^0 \to 2\gamma$ decay will thus be enhanced by the factor $(1+\gamma)^2$, where

$$\gamma \equiv \frac{(m_\pi^2 - m_K^2)}{2(m_\eta^2 - m_K^2)}\left(\frac{m_w}{m_\rho}\right)^2 \frac{(1-\beta)f}{f + \epsilon(g\sin\theta\cos\theta - f\cos^2\theta)}.$$

It is found that the value of γ is quite insensitive to the numerical values of θ and g/f which are discussed in the literature.[26] All give a value around $\gamma \approx -2.3$, from which

$$P(K_2^0 \to 2\gamma) \approx 5.9(1 \pm 0.5) \times 10^4\ \text{sec}^{-1}. \tag{13}$$

The $\omega - \varphi$ mixing effect thus can increase the $K_2^0 \to 2\gamma$ decay rate by an order of magnitude.

The branching ratio of $K_2^0 \to 2\gamma$ decay is thus predicted to be around 0.35% of K_2^0 decay. An experimental check on this point[27] will be enlightening in testing the model discussed in this paper.

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[27] See also J. Dreitlein and H. Primakoff, Phys. Rev. **124**, 268 (1961); N. Cabibbo and E. Ferrari, Nuovo Cimento **18**, 928 (1960).

IV. $K_2{}^0 \to \pi^+ + \pi^- + \gamma$ AND $K^+ \to \pi^+ + \pi^0 + \gamma$ DECAY

As another test of our scheme for the nonleptonic interaction, we discuss in this section the radiative decays $K_2{}^0 \to \pi^+ + \pi^- + \gamma$ and $K^+ \to \pi^+ + \pi^0 + \gamma$. In the following discussion we shall estimate the rates, using the pseudoscalar-meson pole approximation and the SU(3) symmetry:

(a) $K_2{}^0 \to \pi^+ + \pi^- + \gamma$ Decay

According to CP invariance, $K_2{}^0 \to \pi^+ + \pi^-$ decay is not allowed. Thus, there is no internal bremsstrahlung contribution in this case. The CP invariance leads to the expectation that the $M1$ photon amplitude dominates for this decay.[28] We know that the $M1$ photon process, $\eta^0 \to \pi^+ + \pi^- + \gamma$, is one of the main modes of η decay, so that we anticipate that the contribution of the η-meson pole term could be important for the $K_2{}^0 \to \pi^+ + \pi^- + \gamma$ decay. Likewise, we have to consider also the pion pole term. In the following, we estimate these contributions using the vector-meson-dominant model.[23] Note that the diagrams such as those corresponding to

$$K_2{}^0 \to \omega^0 \to \pi^+ + \pi^- + \gamma\,,$$
$$K_2{}^0 \to \rho^0 \to \pi^+ + \pi^- + \gamma\,,$$

do not contribute because of invariance. It is also clear that the dipion resonance cannot enhance the final-state interactions, since the $K_2{}^0 \to \sigma^0 + \gamma$ transition does not take place. The resultant Feynman diagrams are illustrated in Fig. 1. For the $\rho\pi\pi$-type vertices we use the SU(3)-invariant Lagrangian[2]

$$\mathcal{L}_{VPP} = iG_{\rho\pi\pi} \mathrm{Tr}(V_\mu [P, \partial_\mu P])\,, \tag{14}$$

where V_μ is the 3×3 matrix corresponding to the octet of vector mesons. The coupling constant $G_{\rho\pi\pi}{}^2/4\pi$ is approximately 0.50, corresponding to the width ≈ 100 MeV of the ρ meson. For the $\rho\pi\gamma$-type vertices we use the following SU(3) generalized form of interaction:

$$\begin{aligned}\mathcal{L}_{VP\gamma} = i\lambda_{\rho\pi\gamma}\epsilon_{\alpha\beta\gamma\delta}\partial_\alpha A_\beta \{ &\partial_\gamma \pi^0 \rho_\delta{}^0 + \partial_\gamma \pi^+ \rho_\delta{}^- \\ &+ \partial_\gamma \pi^- \rho_\delta{}^+ + \sqrt{3}\partial_\gamma \eta^0 \rho_\delta{}^0 + \partial_\gamma K^- K_\delta{}^{*+} + \partial_\gamma K^+ K_\delta{}^{*-} \\ &- 2\partial_\gamma K^0 \bar{K}_\delta{}^{0*} - 2\partial_\gamma \bar{K}^0 K_\delta{}^{*0} + \cdots \}\,. \end{aligned} \tag{15}$$

The coupling constant $\lambda_{\rho\pi\gamma}$ can be estimated from the $\pi^0 \to 2\gamma$ decay rate. Using the model in which the pion first emits one of the final-state photons and an intermediate ρ meson, which subsequently decays into the other photon, we write the decay rate as

$$P(\pi^0 \to 2\gamma) \simeq \frac{\alpha}{16}\left(\frac{\lambda_{\rho\pi\gamma}}{G_{\rho\pi\pi}}\right)^2 m_\pi{}^3\,,$$

where α is the fine-structure constant.

[28] For the detailed discussion of the final-state interaction in $K \to \pi + \pi + \gamma$ decay, see H. Chew, Nuovo Cimento **26**, 1109 (1962).

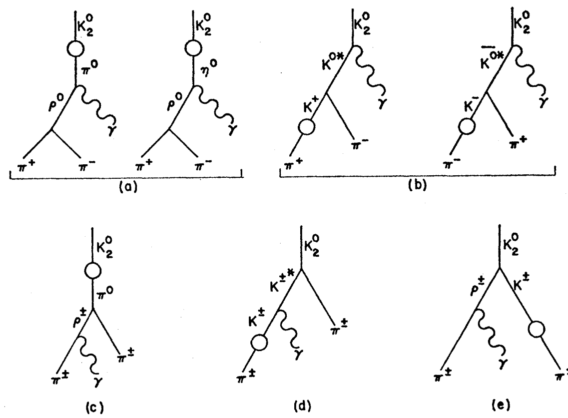

FIG. 1. Feynman diagrams for the $K_2{}^0 \to \pi^+ + \pi^- + \gamma$ decay. The circle denotes the one-pole weak transition. In diagrams (a) and (c), the weak-coupling constant is $f_W(-m_K{}^2)$, while for (b), (d), and (e) $f_W(-m_\pi{}^2)$.

From the rate of $\pi^0 \to 2\gamma$ decay (8), we find

$$m_\pi{}^3 \lambda_{\rho\pi\gamma}{}^2 \approx 1.3 \times 10^{20}\ \mathrm{sec}^{-1}\,. \tag{16}$$

Now the amplitude corresponding to the diagrams of Fig. 1(a) takes the form[13]

$$-4\left(\frac{m_K{}^2}{m_\pi{}^2 - m_K{}^2} + \frac{m_K{}^2}{m_\eta{}^2 - m_K{}^2}\right) f_W \lambda_{\rho\pi\gamma} G_{\rho\pi\pi} \epsilon_{\mu\nu\lambda\beta} \times k_\mu e_\nu P_\lambda{}^+ P_\beta{}^- \frac{1}{m_\rho{}^2 + (p^+ + p^-)^2}\,,$$

where k, p^+, and p^- are respectively the four-momenta of the photon and the π^+ and π^- mesons. e_ν represents the polarization four-vector of the photon. We have here used the exact SU(3) relations for the coupling constants:

$$(f_W)_{K_2{}^0 \to \pi^0} = 3^{-1/2} (f_W)_{K_2{}^0 \to \eta^0}\,,$$

and

$$\sqrt{3}\lambda_{\rho^0\pi^0\gamma} = \lambda_{\rho^0\eta^0\gamma}\,.$$

In a similar way, we can construct all other relevant Feynman amplitudes.[29] It should be noted here that the weak coupling constant f_W is a function of the square of the external momentum q^2. For the diagrams in Figs. 1(a) and 1(c), $q^2 = -m_K{}^2$. For the diagrams in Figs. 1(b), 1(d), and 1(e), $q^2 = -m_\pi{}^2$. In what follows, we shall use f_W to denote $f_W(q^2 = -m_K{}^2)$, and γ to denote the difference

$$\gamma = \frac{f_W(-m_\pi{}^2) - f_W(-m_K{}^2)}{f_W(-m_K{}^2)}\,. \tag{17}$$

[29] In the intermediate states we have neglected the mass difference between ρ and K^* meson and used the exact SU(3) relation (14) and (15). This does not lead to a serious error unless the $KK\rho$ coupling constant turns out to be very different from the predicted value of SU(3) symmetry.

Then, after a lengthy numerical integration, we find

$$P(K_2{}^0\rightarrow\pi^++\pi^-+\gamma)$$
$$=0.51(1+0.53\gamma-0.003\gamma^2)(1\pm0.5)\times10^3\ \mathrm{sec}^{-1},$$

which, assuming that f_W is constant,[30] $\gamma=0$, is

$$P(K_2{}^0\rightarrow\pi^++\pi^-+\gamma)\approx0.51(1\pm0.5)\times10^3\ \mathrm{sec}^{-1}.$$

Thus, we find that $P(K_2{}^0\rightarrow\pi^++\pi^-+\gamma)$ is small. This is due partly to the fact that the contribution of the pion-pole term interferes destructively with that of η-meson pole term, and partly to the momentum-barrier suppression effect for the $M1$ photon. If we include the ω-φ mixing in the γ-$3Ps$ meson vertex, we then obtain a larger value for the decay rate. (See Appendix.)

$$P(K_2{}^0\rightarrow\pi^++\pi^-+\gamma)$$
$$\simeq(2-3)\times(1\pm0.5)\times10^3\ \mathrm{sec}^{-1}. \qquad (18)$$

(b) $K^+\rightarrow\pi^++\pi^0+\gamma$ Decay

Let us next discuss the $K^+\rightarrow\pi^++\pi^0+\gamma$ decay. In this case, we have the internal bremsstrahlung contribution associated with the observed $K^+\rightarrow\pi^++\pi^0$ decay. As regards the direct amplitude we have, of course, the $M1$ photon emission amplitude analogous to the one in $K_2{}^0\rightarrow\pi^++\pi^-+\gamma$ decay discussed above. In addition, we may expect a sizable contribution in this case from the amplitude of the following form:

$$(e_\mu k_\nu-e_\nu k_\mu)P_\mu{}^KP_\nu{}^+F((p^+\cdot k),(P^K\cdot k)). \qquad (19)$$

In the SU(3) octet scheme, the $K^+\rightarrow\pi^++\pi^0$ decay is the process of order e^2, whereas the $K^+\rightarrow\pi^++\pi^0+\gamma$ decay is of order e. Thus, unless the momentum-barrier effect prevents the radiative decay, one may expect that the branching ratio of radiative decay will be rather high. However, the observed ratio, though not comfortably accurate, is very small and in rough agreement with the calculation without direct interactions.[31]

Cabbibo and Gatto[32] have tried to show the smallness of the direct term of the form (19). However, the calculated term gives a vanishing result if we neglect the mass difference of the charged and neutral pion. A plausible model calculation has been performed by Pati,[33] and also by Pepper and Ueda.[34] They both inferred that the contribution of the term (19) would not dominate the internal bremsstrahlung term. [Note that there is an interference between the internal bremsstrahlung term and (19), but the $M1$ term will not interfere with either of them.] Therefore, it is interesting to study the contribution of $M1$ photon emission term. For $K^+\rightarrow\pi^++\pi^0+\gamma$ decay, only the pion-pole term contributes, in contrast to the $K_2{}^0\rightarrow\pi^++\pi^-+\gamma$ case. Consistently, we neglect the vector-meson contribution as we did in the problem of $K_1{}^0$-$K_2{}^0$ mass difference. There are altogether ten corresponding Feynman diagrams similar to Fig. 1 which are destructive. Carrying out a calculation quite similar to that for the $K_2{}^0\rightarrow\pi^++\pi^-+\gamma$ decay, we then obtain for the total $M1$ emission rate (including the effect of ω-φ mixing. See Appendix).

$$P_{M1}(K^+\rightarrow\pi^++\pi^0+\gamma)\simeq(1\pm0.5)\times10^2\ \mathrm{sec}^{-1}. \qquad (20)$$

This value is smaller than the contribution from the internal bremsstrahlung. The experiments by Monti *et al.*[35] for the π^+ energy between 55 and 80 MeV give the branching ratio

$$R=P(K^+\rightarrow\pi^++\pi^0+\gamma)/P(K^+\rightarrow\mathrm{all})\approx8\times10^{-4},$$

while the internal bremsstrahlung alone gives only 1.6×10^{-4} for this ratio. The present estimates of $M1$ photon emission for this range of pion energy give

$$R_{M1}=P(K^+\rightarrow\pi^++\pi^0+\gamma)_{M1}/P(K^+\rightarrow\mathrm{all})\approx1\times10^{-6}.$$

Thus, we do not expect that the $M1$ photon emission mechanism considered here does increase the rate of $K^+\rightarrow\pi^++\pi^0+\gamma$ decay significantly, and the rate will be essentially dominated by the internal bremsstrahlung contribution. We certainly need better experimental statistics before drawing any definite conclusion from experiments.[36] It is, however, clear that the observed small rate of $K^+\rightarrow\pi^++\pi^0+\gamma$ decay does not present any contradiction to the $|\Delta I|=\frac{1}{2}$ rule. We hope that a systematic study of $K^+\rightarrow\pi^++\pi^0+\gamma$ decay will soon settle the problem.

V. COMMENT ON $K\rightarrow3\pi$ DECAY

In the preceding discussion, we have used the value of the coupling constant f_W given by (4) estimated from the $K_1{}^0$-$K_2{}^0$ mass difference. We now consider the same approximation for the $K\rightarrow3\pi$ decay.

(a) Pion-Pole Approximation in $K\rightarrow3\pi$ Decay

We write the total amplitude in $K\rightarrow3\pi$ decay in the following form, using the linear matrix-element theory in which the denominators of Feynman propagators are approximated as constant:

$$M_k=C_k[1+\alpha_k(S_3-S_0)/m_\pi{}^2]. \qquad (21)$$

[30] Unless there is a strong momentum dependence of $f_W(q^2)$, which is not likely, the value (18) would not change significantly.
[31] J. Good, Phys. Rev. **113**, 352 (1959).
[32] N. Cabibbo and R. Gatto, Phys. Rev. Letters **5**, 382 (1960).
[33] J. C. Pati (private communication). He used baryon loop model.
[34] S. V. Pepper and Y. Ueda (to be published).
[35] D. Monti, G. Quareni, and A. Quareni Vinudelli, Nuovo Cimento **21**, 550 (1961).
[36] Our results on the rates of $K_2{}^0\rightarrow\pi^++\pi^-+\gamma$ and $K^+\rightarrow\pi^++\pi^0+\gamma$ decays seems to be smaller by an order of magnitude than the corresponding values given in the Ref. 34. This is due partly to the fact that our value $\rho\pi\gamma$ coupling constant (based on the vector-meson dominant model for $\pi^0\rightarrow2\gamma$ decay) is smaller than the value used in Ref. 34. It may also be due to the fact that in Ref. 34, the contributions of the diagrams of the type Figs. 1(b), 1(d), and 1(e), which should be considered in the SU(3) symmetry on an equal footing, are not included. We would like to thank Dr. C. Kacser and Dr. P. Singer for pointing out the possible importance of the Feynman diagrams which were not considered in Ref. 13.

k specifies whether the decay is in the τ or τ' mode or $K_2{}^0$ modes. $S_i=-(q-p_i)^2$ and q and p_i are the four momenta of the kaon and pions, respectively. p_3 stands for the unlike pion (in the case of $K_2{}^0$ decay, the π^0 meson). S_0 is the symmetric point defined by

$$S_0=m_\pi{}^2+\tfrac{1}{3}m_K{}^2.$$

C_k denotes the totally symmetric constant amplitude which dominates the decay rate. The data collected by Berley *et al.*[37] indicate for the values of the asymmetry parameter

$$\alpha_\tau\approx-0.089,\quad \alpha_{\tau'}\approx 0.18. \tag{22}$$

Let us first discuss the constant amplitude. In the eightfold way, we have two diagrams shown in Fig. 2. We use the SU(3)-invariant Lagrangian

$$\mathfrak{L}=4\pi\lambda[\boldsymbol{\pi}\boldsymbol{\pi}+\eta^{0\,2}+\bar{K}K+\bar{K}^0K^0]^2 \tag{23}$$

for the four-point interaction of the PS meson with the coupling constant λ. Then from Figs. 2(a) and 2(b), we obtain

$$C_\tau=2C_{\tau'}=-64\lambda \times[f_W(-m_K{}^2)-f_W(-m_\pi{}^2)]\frac{m_K{}^2}{m_K{}^2-m_\pi{}^2}. \tag{24}$$

Here we have taken into account the momentum dependence of the vertex f_W as in (17). If this momentum dependence is small, as we usually expect, the contributions of the two diagrams tend to cancel each other as is seen in (24). This appearance of dynamical suppression for the otherwise most contributing diagrams is, in fact, a rather welcome situation in our dynamical model.[38] From (24), the τ-decay rate can be calculated as

$$P(K^+\to\pi^++\pi^-+\pi^+) =80\times|\gamma f_W(-m_K{}^2)|^2|\lambda|^2\text{ MeV}, \tag{25}$$

where γ is again given by (17).

With the value of $f_W(-m_K{}^2)$ given by (4), and the value $\lambda=-0.18\pm0.05$ deduced by Hamilton *et al.*,[39] the comparison with the observed rate indeed indicates the need of suppression of the order $|\gamma|^2\approx 1/15$. Since, as shown above, we can reasonably expect the occurrence of the suppression[40] of the constant amplitudes (Fig. 2), our estimate of $f_W(-m_K{}^2)$ in (4) does not lead to any contradiction as far as the rate is concerned. However, in order to obtain a consistent dynamics, we have also to check the asymmetric components. Since we

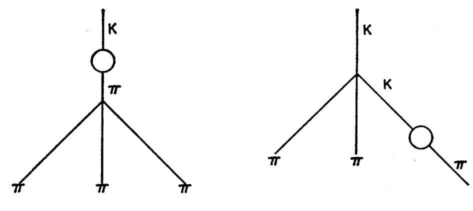

FIG. 2. Feynman diagrams for the symmetric constant amplitudes of $K\to3\pi$ decay in the pion-pole approximation.

expect a suppression of the constant amplitude, we have to worry about the possibility in our model that the asymmetry parameter becomes too large. Now we first discuss the asymmetric components by assuming that the P-wave final-state interaction due to vector-meson resonance states dominates the processes under consideration.[41] Hori *et al.*[38] have shown that in the one-pion-pole approximation, if we include all the possible contributions due to vector mesons (ρ and K^*) which are allowed in the eightfold way, the asymmetric components themselves also tend to cancel each other (they again are proportional to $[f_W(-m_K{}^2)-f_W(-m_\pi{}^2)]$ in the limit of SU(3) symmetry and of the momentum independence of f_W. In fact, if we keep this momentum dependence, the asymmetry parameter can be expressed as[42]

$$\alpha_\tau=-\tfrac{1}{2}\alpha_\tau{}'=\alpha_\rho\frac{1}{[f(-m_K{}^2)-f(-m_\pi{}^2)]} \times\left[f(-m_K{}^2)-f(-m_\pi{}^2)\left(\frac{1}{2}\frac{G_{\rho KK}}{G_{\rho\pi\pi}} +\frac{1}{2}\frac{G_{K^*K\pi}{}^2}{G_{\rho\pi\pi}{}^2}\frac{m_\rho{}^2-S_0}{m_{K^*}{}^2-S_0}\right)\right], \tag{26}$$

where

$$\alpha_\rho\equiv\frac{3}{4\lambda}\left(\frac{G_{\rho\pi\pi}{}^2}{4\pi}\right)\frac{m_\pi{}^2}{m_\rho{}^2-\frac{1}{3}m_K{}^2-m_\pi{}^2}\approx-0.085. \tag{27}$$

Now, if we insert the observed mass of the K^* and ρ meson, and the values of the vector-meson coupling constants $G_{K^*K\pi}$ and $G_{\rho\pi\pi}$ with which the experimental widths of the K^* and ρ mesons are best fitted, we obtain

$$\frac{G_{K^*K\pi}{}^2}{G_{\rho\pi\pi}{}^2}\frac{m_\rho{}^2-S_0}{m_{K^*}{}^2-S_0}\approx 1. \tag{28}$$

Then, if the SU(3) prediction

$$G_{\rho KK}=G_{\rho\pi\pi} \tag{29}$$

[37] For instance, see D. Berley, D. Colley, and J. Schultz, Phys. Rev. Letters **20**, 114 (1963).

[38] S. Hori, S. Oneda, S. Chiba, and A. Wakasa, Phys. Letters **5**, 339 (1963).

[39] J. Hamilton, P. Menotti, G. Dades, and L. Vick, Phys. Rev. **128**, 1881 (1962).

[40] $|\gamma|^2$ could be smaller than 1/15. In this case, we can imagine that contributions other than the pion-pole diagram also contribute to the constant amplitude appreciably so as to obtain the observed rate. This is natural, since in such an approximation we usually assume that the background effect is around 10% or less.

[41] Such a possibility has been considered by M. A. Bagi Beg and P. C. DeCelles, Phys. Rev. Letters **8**, 46 (1962); Riazuddin and Fayyazudin, *ibid.* **7**, 464 (1961); G. Barton and S. P. Rosen, *ibid.* **8**, 414 (1962); C. Kacser, Phys. Rev. **132**, 339 (1963).

[42] S. Oneda and Y. S. Kim, University of Maryland, Technical Report No. 357, 1964 (unpublished).

holds, we obtain from (26) and (27)

$$\alpha_\tau \approx \alpha_\rho \approx -0.086\,, \tag{30}$$

which surprisingly well reproduces the experimental value [Eq. (22)].

Thus, we have shown that in our model both the constant amplitudes and the asymmetric components receive the same suppression effect due to SU(3) symmetry; and the relative ratio of their magnitudes [i.e., α_τ, Eq. (30)] does not exceed the observed value and, as a matter of fact, is close to the observed one. We have demonstrated above that there will be no unusual enhancement of the asymmetric parameter corresponding to the suppression of constant amplitude. *In this respect, our model does not show any inconsistency with regard to either the rate or asymmetric components.*

Now, if we assume that the nonleptonic interactions have an origin different from the usual current-current picture[43] (in contrast to the leptonic processes), the above discussion may provide an attractive explanation of the observed sign and magnitude of asymmetry parameter of $K \to 3\pi$ decay.[42] However, if we insist on the current-current interactions, the above discussion on the asymmetric components does not exhaust the whole story. We have to expect an asymmetric contribution also from the intrinsic vector nature of the primary weak interaction. We write the strangeness-violating basic interaction in the form

$$= J_\alpha S_\alpha{}^\dagger + \mathrm{H.c.}\,, \tag{31}$$

where J_α and S_α, respectively, denote the strangeness-conserving and -nonconserving currents; and they, of course, consist of vector and axial-vector currents. Now let us, for instance, look at the following matrix element, which is obtained by a simple factorization of the vector part of current-current interaction:

$$\langle \pi^+\pi^0 | J_\alpha{}^V | 0\rangle \langle \pi^0 | S_\alpha{}^{V\dagger} | K^+\rangle . \tag{32}$$

This matrix element can contribute to the asymmetric component of τ' decay, whose magnitude is calculable, since $\langle \pi^0 | S_\alpha{}^{V\dagger} | K^+\rangle$ is the matrix element of $K^+ \to \pi^0 + e^+ + \nu$ decay, and $\langle \pi^+\pi^0 | J_\alpha{}^V | 0\rangle$ is that of $\pi^+ \to \pi^0 + e^+ + \nu$ decay. It turns out that the magnitude of $\alpha_{\tau'}$ calculated from this intrinsic structure of weak interactions can contribute as much as 60% (sign is unknown) of the observed value.[44] From the neutral vector currents (which are supposed to be there from the strict $|\Delta I| = \frac{1}{2}$ rule), we also obtain $\alpha_\tau (\alpha_{\tau'} = -2\alpha_\tau)$.

Thus, we have demonstrated that the intrinsic vector nature of basic interactions alone can give rise to a sizable amount of asymmetric components of $K \to 3\pi$ decay. The actual situation is more complicated, since both the final-state interactions due to the vector resonances and the intrinsic mechanism will operate in the current-current picture of weak interaction. We shall content ourselves with noting that a consistent dynamical picture could be obtained, since the suppression factor γ can be regarded, at the moment, as an adjustable parameter.[40]

Finally, we shall add a comment on the similar decay $\eta \to 3\pi$. Hori *et al.*[38] have also shown that a similar cancellation of the constant amplitude takes place in this case. However, for the asymmetric components there exists no cancellation in the vector-meson contributions, unlike the $K \to 3\pi$ decay case discussed above, so that we should expect a larger P-wave contribution in the $\eta^0 \to \pi^+ + \pi^- + \pi^0$ case. (We note here that there are no intrinsic asymmetry components in $\eta \to 3\pi$ decay, in contrast to the $K \to 3\pi$ case, since $\eta \to \pi + \gamma$ transition is forbidden.) Recent experiments seem to indicate that this is the case.[45]

Brown and Singer[46] have recently shown that the existence of the dipion σ^0 could explain the asymmetry and branching ratio of $\eta \to 3\pi$ and $K \to 3\pi$ decays. We would like to say a word about a connection between their model and ours. If the dipion really exists, it would dominate the low-energy behavior of the pion-pion scattering. This would force us to redefine the coupling constant λ of (23) in terms of the masses and widths of the dipion[17,18] and its SU(3) counterparts. In our one-pole approximation we can accommodate this situation by inserting the propagators of the dipion and its SU(3) counterparts in the vertices where Eq. (23) is operating in Fig. 2. It is then easy to see that if the dipion belongs to the SU(3) singlet, the same cancellation occurs as discussed before. Therefore, our conclusion would not change. If, however, the dipion belongs to a higher multiplet, then we may have to consider the mass splitting among the multiplet members. For instance, if the σ^0 meson with its mass 390 MeV and width 75 MeV is the $I = Y = 0$ member of an octet, and if other members have much masses, then the Brown-Singer effect would actually take place as a consequence of this violation of the SU(3) symmetry in our model for both the η and K decays.[47] However, we also note that the assumption is not easily justified that the dipion effect is *entirely* responsible for the asymmetric component of $K \to 3\pi$ decay, since the intrinsic current-current nature of the weak interaction itself can lead to a sizable asymmetry in $K \to 3\pi$ decay.

(b) Possible Violation of $|\Delta I| = \frac{1}{2}$ Rule in the $K_2{}^0 \to 3\pi$ Decay

We note that the $\eta \to 3\pi$ decays are electromagnetic processes, and compete favorably with the $\eta \to 2\gamma$ or $\eta \to \pi^+ + \pi^- + \gamma$ decays. Thus, if we take the standpoint

[43] For instance, see R. E. Marshaki, C. Ryan, T. K. Radah, and K. Raman, Phys. Rev. Letters **11**, 396 (1963).

[44] J. C. Pati and S. Oneda, Phys. Rev. **136**, B1097 (1964).

[45] S. Oneda, Y. S. Kim, and L. M. Kaplan, Nuovo Cimento (to be published).

[46] F. C. Crawford, Jr., L. Lloyd, and E. Fowler, Phys. Rev. Letters **10**, 546 (1963) and **11**, 564 (1963).

[47] L. M. Brown and P. Singer, Phys. Rev. Letters **8**, 460 (1962) and Phys. Rev. **133**, B812 (1964).

that the violation of the $|\Delta I|=\frac{1}{2}$ rule is strictly due to the electromagnetic interaction, the η-meson pole contribution is the most likely candidate for the violation of $|\Delta I|=\frac{1}{2}$ rule in the $K_2^0 \to 3\pi$ decay. We present here a crude estimate of[48] the strength of this violation based on the SU(3) symmetry and available experimental data on the η decays. For simplicity, we assume here that the η decay is dominated by the symmetric constant amplitude,[49] and we determine the magnitude of this amplitude from the total decay rate. Then the deviation from the $|\Delta I|=\frac{1}{2}$ rule in the $K_2^0 \to 3\pi$ decay rates can be expressed as[50]

$$P(K_2^0 \to \pi^+ + \pi^- + \pi^0) = (1-x)^2 P_{|\Delta I|=1/2}(K_2^0 \to \pi^+ + \pi^- + \pi^0), \quad (33)$$

$$P(K_2^0 \to \pi^0 + \pi^0 + \pi^0) = (1-y)^2 P_{|\Delta I|=1/2}(K_2^0 \to \pi^0 + \pi^0 + \pi^0), \quad (34)$$

where

$$P_{|\Delta I|=1/2}(K_2^0 \to \pi^+ + \pi^- + \pi^0) = (2.87 \pm 0.23) \times 10^6 \text{ sec}^{-1},$$

$$P_{|\Delta I|=1/2}(K_2^0 \to \pi^0 + \pi^0 + \pi^0) = (5.55 \pm 0.44) \times 10^6 \text{ sec}^{-1},$$

are the rates predicted from the observed $P(K^+ \to \pi^+ + \pi^- + \pi^+)$ and $P(K^+ \to \pi^+ + \pi^0 + \pi^0)$, using the $|\Delta I|=\frac{1}{2}$ rule.[51]

The parameters x and y are related to the strength of the η-meson pole contribution and to the rates of η decay by

$$x^2 = 8.1 \times 10^9 |f_{K^0\eta^0}|^2 \{P(\eta^0 \to \pi^+ + \pi^- + \pi^0) \text{ in eV}\}, \quad (35)$$

$$y^2 = 8.1 \times 10^9 |f_{K^0\eta^0}|^2 \{P(\eta^0 \to \pi^0 + \pi^0 + \pi^0) \text{ in eV}\}, \quad (36)$$

where $f_{K^0\eta^0}$ is the $K^0-\eta^0$ coupling, and is $3^{-1/2}$ of f_W if we use the octet scheme discussed in Sec. II. Unfortunately, we do not have experimental values of the $\eta \to 3\pi$ decay rates. We thus make a theoretical estimate based on the SU(3) symmetry. From the largest decay rate of the $\pi^0 \to 2\gamma$ decay so far reported, $P(\pi^0 \to 2\gamma) \approx 6$ eV, we obtain $P(\eta^0 \to 2\gamma) \approx 140$ eV using the SU(3) symmetry. From the branching ratios[46] of η decays, we have a rough estimate:

$$P(\eta^0 \to \pi^+ + \pi^- + \pi^0) \approx 112_{-40}^{+112} \text{ eV}, \quad (37)$$

$$P(\eta^0 \to \pi^0 + \pi^0 + \pi^0) \approx 93_{-57}^{+165} \text{ eV}. \quad (38)$$

Thus, with the value f_W in (4), we obtain

$$x^2 \approx (0.90_{-0.32}^{+0.90}) \times (1 \pm 0.5) \times 10^{-2}, \quad (39)$$

$$y^2 \approx (0.75_{-0.46}^{+1.33}) \times (1 \pm 0.5) \times 10^{-2}. \quad (40)$$

Recently, Stern *et al.*[52] reported the rate

$$P(K_2^0 \to \pi^+ + \pi^- + \pi^0) = (2.90 \pm 0.72) \times 10^6 \text{ sec}^{-1},$$

which is in rather good agreement with the strict $|\Delta I|=\frac{1}{2}$ rule. It should be noted, however, that the experimental error is still large. It should also be noted that the measurements on the $\pi^0 \to 2\gamma$ rate, the K_1^0-K_2^0 mass difference, and the $\eta \to 3\pi$ branching ratios are still controversial (within a factor 2 to 3), and that this can cause a still larger change in the value of the parameters x and y. Thus, we hope that these experiments will be comfortably settled in the near future. If we use tentatively the values $\Delta m(K^0) \approx 0.65 \times 10^{-11}$ MeV and $P(\eta^0 \to \pi^+ + \pi^- + \pi^0) \approx 112$ eV we obtain $|x| \approx 0.1$, which is not inconsistent with the present experimental data.[53] If it turned out that the violation of the rule is indeed very small, then it will be a good indication either (a) that the numerical value of $f_{K^0\eta^0}$ is smaller than what we would expect from the SU(3) octet model of the weak transition, or (b) that some other effect like the contribution of the unitary singlet $I=Y=0$ pseudoscalar meson η' becomes important.

In case (a), the contribution of the η-meson term in the K_1^0-K_2^0 mass difference will be small, and as a consequence, the pion term becomes important, so that we expect $f_{K^0\eta^0} \ll f_{K^0\pi}$ and

$$m(K_1^0) < m(K_2^0), \quad (41)$$

contrary to Eq. (5). For a general discussion of this problem without assuming the SU(3) octet model, see Ref. 12.

VI. CONCLUDING REMARKS

In this paper we have attempted to look for the experimetal consequence of the SU(3) octet scheme for the nonleptonic K-meson processes. We shall summarize our main results and add a few further comments.

(a) $K_1^0 - K_2^0$ Mass Difference

As a way of estimating the weak-coupling constant f_W in the PS-meson pole approximation, we have used the experimental value of K_1^0-K_2^0 mass difference. The necessary consequences of this procedure were the estimate of f_W given in (4) and the sign of the mass difference (5) [i.e., $m(K_1^0) > m(K_2^0)$]. We have remarked that the dipion will not make a contribution in the Gell-Mann–Cabibbo model.[19] We also inferred that the vector-meson contribution is negligibly small.[20] As regards the magnitude of f_W, we may encounter an objection that our value of f_W is too large. Perhaps such

[48] However, this does not mean at all that the vector meson ρ contribution to the asymmetric component estimated in Ref. 45 is negligible.

[49] For more general treatment (without assuming the octet model of nonleptonic interaction), see Ref. 12.

[50] The $\eta^0 \to \pi^+ + \pi^- + \pi^0$ decay seems to have a rather large asymmetry, but this will not produce a serious error in the following argument.

[51] See for instance, R. H. Dalitz, Brookhaven National Laboratory Report No. 837, 1963 (unpublished).

[52] D. Stern, T. Binford, V. Lind, J. Anderson, F. Crawford, Jr., and R. Golden, Phys. Rev. Letters **12**, 459 (1964). Earlier references are cited there where the discrepancy from $|\Delta I|=\frac{1}{2}$ rule was larger.

[53] We feel that in the present model the violation of the $|\Delta I|=\frac{1}{2}$ rule more or less of this order is rather expected, unless other contributions (for instance, the unitary singlet $I=Y=0$ η'-meson contribution) are important.

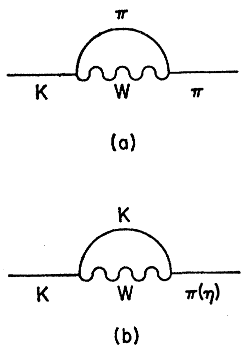

FIG. 3. Feynman diagrams which may dominate the weak $K-\pi(\eta)$ transition mechanisms. W denotes the intermediate vector meson.

an objection will arise if we make an estimate of the $K-\pi$ vertex by using a factorization approximation for the axial-vector currents $\langle\pi^+|J_\alpha{}^A|0\rangle\,\langle 0|S_\alpha{}^A|K^+\rangle$. Since $\langle\pi^+|J_\alpha{}^A|0\rangle$ and $\langle 0|S_\alpha{}^{A+}|K^+\rangle$ are the form factors of $\pi\to\mu+\nu'$ and $K\to\mu+\nu'$ decay, respectively, we can evaluate this term and obtain[17]

$$f_W(-m_K{}^2)\approx 7.4\times 10^{-16}\,, \tag{42}$$

which is certainly smaller than the value obtained in (4). However, we do not think that this term dominates the real $K-\pi$ transition. Here we recall the discussion of Sec. V for the $K\to 3\pi$ decay, where we have met with a similar example. There we discussed the term $\langle\pi^+\pi^0|J_\alpha{}^V|0\rangle\,\langle 0|S_\alpha{}^{V\dagger}|\pi^0K^+\rangle$ obtained from a similar factorization procedure. However, this term only contributes to the asymmetric components, and its contribution to the decay rate is negligibly small. Thus, it is not surprising that the term $\langle\pi^+|J_\alpha{}^A|0\rangle\langle 0|S_\alpha{}^{\dagger A}|K^+\rangle$ also contributes only a few percent to the actual $K-\pi$ coupling.[54] As a possible mechanism which dominates the $K-\pi(\eta)$ transitions, we may mention the Feynman diagrams shown in Figs. 3(a) and 3(b). W denotes the intermediate vector meson. The coupling constant at each vertex of Fig. 3 is determined if we assume that J_α and S_α belong to octet from the known leptonic decays and SU(3) symmetry. We anticipate that this will lead to a larger value of f_W than (14).

Now we turn to the sign of the mass difference. In Eq. (3), the sign is determined by the difference of two terms which are of a comparable order of magnitude, and it is sensitive to the ratio $f_{K^0\eta^0}/f_{K^0\pi^0}$. The unitary singlet PS meson (which we call η' in this paper) could contribute if it exists, and its contribution to $\Delta m(K^0)$ will be positive or negative according to $m_{\eta'}>m_K$ or $m_{\eta'}<m_K$, respectively. The present experimental evidence[55] indicates $m_{\eta'}\approx 960$ MeV, so that its contribution is positive. Therefore, its existence favors more the case $m(K_1{}^0)>m(K_2{}^0)$, and its inclusion in our discussion of Sec. II will lead to a slightly smaller value of f_W than (4). Of course, since η' is heavy, its contribution will be relatively unimportant unless $f_{K^0\eta'}$ is very large.

On the other hand, if we really want to obtaim $m(K_1{}^0)<m(K_2{}^0)$ in our model, we probably have to look for the effect of the violation of SU(3) symmetry, unless the vector-meson intermediate states are important.[56] For instance, let us look at Fig. 3. The mass splitting between the intermediate particles π, K, and η could violate the SU(3) symmetry relation given by (2). If, however, the W-meson mass is much larger than the mass splitting, the violation may not be so large as to induce the change in the sign of mass difference.

Perhaps we cannot really argue about the sign until we have a reasonable understanding of the effect of symmetry-breaking interactions.[57]

(b) *K*-Meson Decays

We have calculated the $K_2{}^0\to 2\gamma$ decay rate

$$P(K_1{}^0\to 2\gamma)\approx 0.76(1\pm 0.5)\times 10^4\ \mathrm{sec}^{-1}\,,$$

which is enhanced by the $\omega-\varphi$ mixing effect to

$$P(K_2{}^0\to 2\gamma)\approx 5.9(1\pm 0.5)\times 10^4\ \mathrm{sec}^{-1}.$$

If we adopt the latter value,[58]

$$\frac{P(K_2{}^0\to 2\gamma)}{(K_2{}^0\to \mathrm{all})}\approx 3.5\times(1\pm 0.5)\times 10^{-3}.$$

We hope that this ratio can be checked experimentally in the future.

Our value is smaller than expected[59] owing to the destructive interference between the π- and η-meson pole terms. Nearly the same interference occurs for the case of $K\to\pi+\pi+\gamma$ decay. We have obtained the decay rate (with ω-φ mixing)

$$P(K_2{}^0\to\pi^++\pi^-+\gamma)\approx(2-3)\times(1\pm 0.5)\times 10^3\ \mathrm{sec}^{-1}$$

so that

$$P(K_2{}^0\to\pi^++\pi^-+\gamma)/P(K_2{}^0\to 2\gamma)\simeq\frac{1}{20}\sim\frac{1}{30}$$

in our model.

We concluded that the contribution of the direct amplitudes in $K^+\to\pi^++\pi^0+\gamma$ decay will be smaller than

[54] The terms obtained by factorizing the currents have a momentum barrier, so that it will be dominated by other less momentum-dependent terms.

[55] G. R. Kalbfleisch, L. Alvarez, A. Barbaro-Galtieri, O. Dahl, P. Eberhard *et al.*, Phys. Rev. Letters **12**, 527 (1964); M. Goldberg, M. Gundzik, S. Lichtman, J. Leitner, M. Primer *et al.*, *ibid.* **12**, 546 (1964).

[56] As mentioned in footnote 20, we are not confident of this possibility.

[57] One may speculate that there may be a rather large η-η' mixing (as in the case of $\omega-\varphi$ mixing) which could change the relation (2). However, there is, at the moment, no compelling reason to expect a large $\eta-\eta'$ mixing in contrast to the case of $\omega-\varphi$ mixing.

[58] We have used 1.7×10^7 sec^{-1} for the lifetime of $K_2{}^0$ meson. J. V. Jovanovich, T. Fujii, F. Turkot, R. W. Burris, D. S. Loebbaka, and G. T. Zorn, Brookhaven National Laboratory Report No. 837, 1963 (unpublished). D. Luers, I. Mittra, W. Willis, and S. Yamamoto, Phys. Rev. **133**, B1276 (1964). Other references are cited here.

[59] C. Bouchiat, J. Nuyts and J. Prentki, Phys. Letters **3**, 156 (1962).

the internal bremmstrahlung amplitude, so that the branching ratio of this decay will be essentially given by the latter term.[60] We have shown that for the $K\to 3\pi$ decays, our value of f_W and the PS-meson pole approximation do not show any inconsistency with respect to either the constant or the asymmetric amplitudes. We have stressed that the more accurate measurement on the $K_2^0\to 3\pi$ decays will be useful in making an estimate of the value of $f_{K^0\eta^0}$.

In the end, we would like to remark that there is still a possibility that the unitary singlet η' may also play a significant role in $K_2^0\to 2\gamma$, $\pi^++\pi^-+\gamma$, and $\pi+\pi+\pi$ decays. In the $K_2^0\to 2\gamma$ and $\pi^++\pi^-+\gamma$ decay, we encountered destructive interference, so that the η' contribution could become relatively important. Since the mass of the η' is rather high, we may as well hope that its effect will not affect our results appreciably. The experimental test of the results of our dynamical model may be interesting also from this standpoint.

APPENDIX: THE EFFECT OF ω-φ MIXING ON THE $K\to\pi+\pi+\gamma$ DECAY

For the $\gamma-3Ps$ meson vertices,[61] we use the vector-meson dominant model in which the $\gamma-3Ps$ interaction is derived from the VPP and $VP\gamma$ interactions. Then, for the matrix element of $K_2^0\to\pi^++\pi^-+\gamma$ decay, we obtain

$$4(2)^{1/2}f_W\left(\frac{m_K^2}{m_K^2-m_\pi^2}\right)\lambda_{\rho\pi\gamma}G_{\rho\pi\pi}\epsilon_{\mu\nu\lambda\beta}k_\mu e_\nu p_\lambda^+p_\beta^-\frac{1}{m_\rho^2}\times\left\{(\gamma-2)+\frac{1}{1-\lambda\cos2\theta+(g/f)\lambda\sin\theta}\times\left[(1+\gamma)\left(-1+\lambda\cos2\theta+\frac{2g}{f}\lambda\sin2\theta\right)\frac{m_\rho^2}{m_{K^*}^2}-\frac{m^2}{m_\rho^2}\left(\frac{m_K^2-m_\pi^2}{m_K^2-m_\eta^2}\right)\right]\right\},$$

and similarly for the $K^+\to\pi^++\pi^0+\gamma$ decay

$$2(2)^{1/2}f_W\left(\frac{m_K^2}{m_K^2-m_\pi^2}\right)\lambda_{\rho\pi\gamma}G_{\rho\pi\pi}\epsilon_{\mu\nu\lambda\beta}k_\mu e_\nu p_\lambda^+p_\beta^0\frac{1}{m_\rho^2}\times\left\{(1-5\gamma)-\frac{1+\gamma}{1-\lambda\cos2\theta+(g/f)\lambda\sin2\theta}\left(\frac{m_\rho^2}{m_{K^*}^2}\right)\times\left[3\frac{m^2}{m_\rho^2}-2(1-\lambda\cos2\theta)+\frac{4g}{f}\lambda\sin2\theta\right]\right\},$$

where θ is the ω-φ mixing angle. All other quantities are defined in Secs. III and IV. In the above expression, we have neglected the momentum dependence of the vector meson propagator, which is not so important in the energy region under consideration. In the limit of exact symmetry (the same mass for the vector mesons and $\theta=0$), and in the approximation of constant $fw(\gamma=0)$, the above matrix element for the $K^+\to\pi^++\pi^0+\gamma$ decay vanishes and that of the $K_2^0\to\pi^++\pi^-+\gamma$ decay is proportional to $(4m_K^2-3m_\eta^2-m_\pi^2)$ which will also vanish if we use the Gell-Mann–Okubo's mass formula. Thus, in the exact symmetry limit, these decays are forbidden in the present model. Thus, the nonzero decay rates are lately due to the symmetry breaking. As in the case of $K_0^2\to 2\gamma$ decay, the results are insensitive to the choice of mixing angle θ and the g/f ratio. We obtain

$$P(K_2^0\to\pi^++\pi^-+\gamma)\simeq(2\text{–}3)\times(1\pm0.5)\times10^3\ \text{sec}^{-1},$$
$$P(K^+\to\pi^++\pi^-+\gamma)\simeq(1\pm0.5)\times10^2\ \text{sec}^{-1}.$$

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[60] The recent experiment on the $K^+\to\pi^++\pi^0+\gamma$ decay seems to be consistent with the present result. See D. Cline and W. F. Fry, Phys. Rev. Letters **13**, 101 (1964).

[61] This Appendix was added in proof.